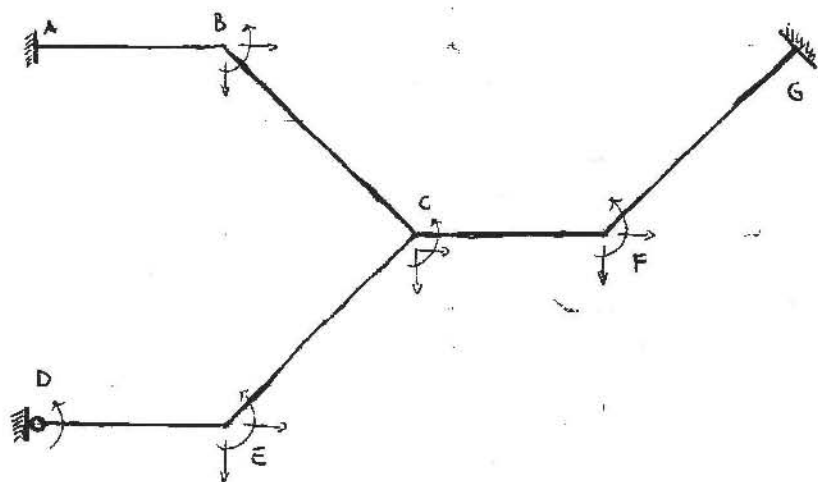


ESERCIZI ANALISI CINEMATICHE FONDAMENTALI

1.



13 parametri incogniti

8 Spost
5 Rotaz.

- Ipotesi di Indeformentilità assiale [5 aste \rightarrow 6 vincoli sugli spostamenti]

$$(\Delta e)_{AB} = 0 \rightarrow \boxed{u_B = 0}$$

$$(\Delta e)_{DE} = 0 \rightarrow \boxed{u_E = 0}$$

$$(\Delta e)_{FG} = 0 \rightarrow -\frac{u_F}{\sqrt{2}} + \frac{w_F}{\sqrt{2}} = 0 \rightarrow [u_F = w_F]$$

$$(\Delta e)_{CF} = 0 \rightarrow -u_C + u_F = 0 \rightarrow [u_C = u_F]$$

$$(\Delta e)_{BC} = 0 \rightarrow -\frac{w_B}{\sqrt{2}} + \frac{u_C}{\sqrt{2}} + \frac{w_C}{\sqrt{2}} = 0 \rightarrow [w_B = u_C + w_C]$$

$$(\Delta e)_{EC} = 0 \rightarrow \frac{w_E}{\sqrt{2}} - \frac{u_C}{\sqrt{2}} + \frac{u_C}{\sqrt{2}} = 0 \rightarrow [w_C = w_E + u_C]$$

$$\boxed{u_C = \delta_1}$$

$$\boxed{u_F = \delta_1}$$

$$\boxed{w_F = \delta_1}$$

$$\boxed{w_C = \delta_2}$$

$$\boxed{w_B = \delta_1 + \delta_2}$$

$$\boxed{w_E = \delta_2 - \delta_1}$$

Riepilogando:

$$\begin{cases} u_B = 0 \\ w_B = \delta_1 + \delta_2 \end{cases}, \begin{cases} u_E = 0 \\ w_E = \delta_2 - \delta_1 \end{cases}, \begin{cases} u_C = \delta_1 \\ w_C = \delta_2 \end{cases}, \begin{cases} u_F = \delta_1 \\ w_F = \delta_1 \end{cases}$$

- Condensazione Rotazione in D

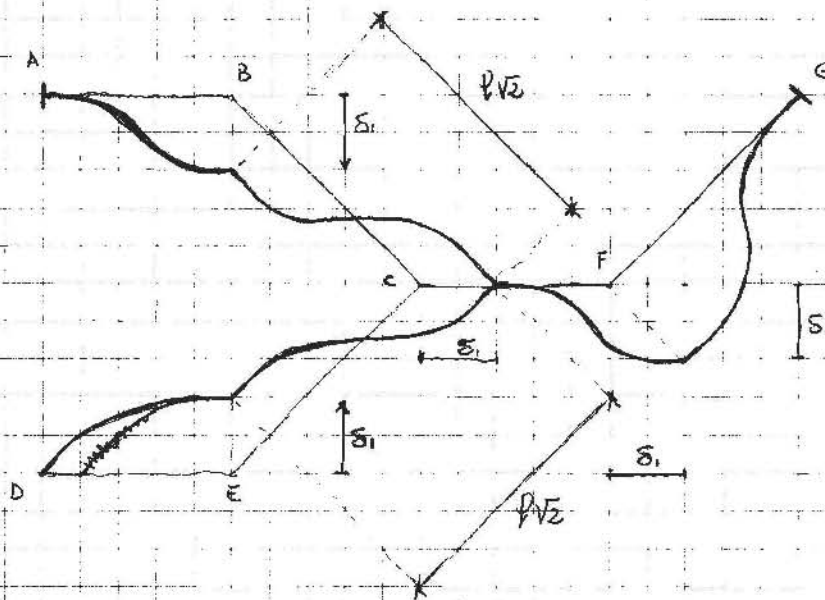
$$\varphi_D = \frac{1}{2} \varphi_E \rightarrow \text{Le rotazioni indipendenti ce sono 4.}$$

- Conclusione

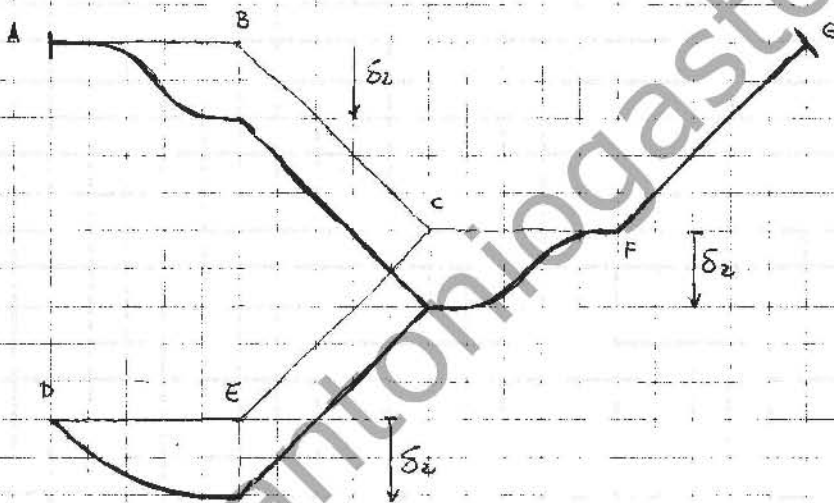
I parametri cinematici indipendenti sono 6 $\rightarrow \begin{cases} 2 \text{ Spostam. } [\delta_1, \delta_2] \\ 4 \text{ Rot. } [\varphi_B, \varphi_C, \varphi_E, \varphi_F] \end{cases}$

- Cinematiche Fondamentali

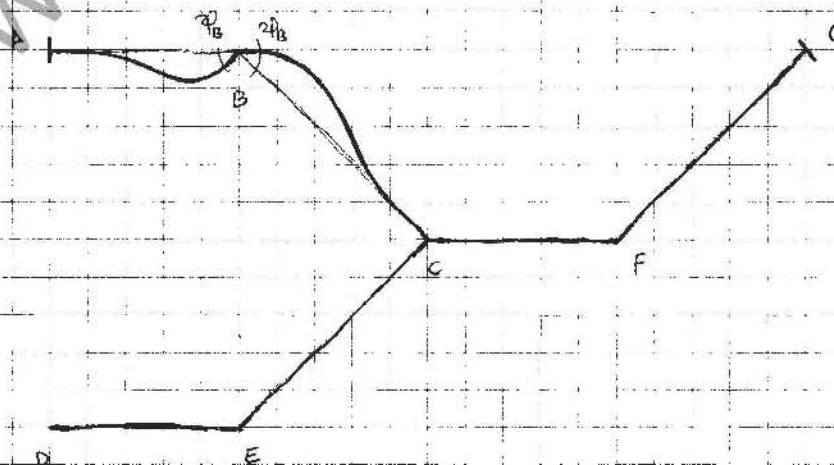
- $[\delta_1 \neq 0, \delta_2 = \varphi_B = \varphi_C = \varphi_E = \varphi_F \neq 0]$



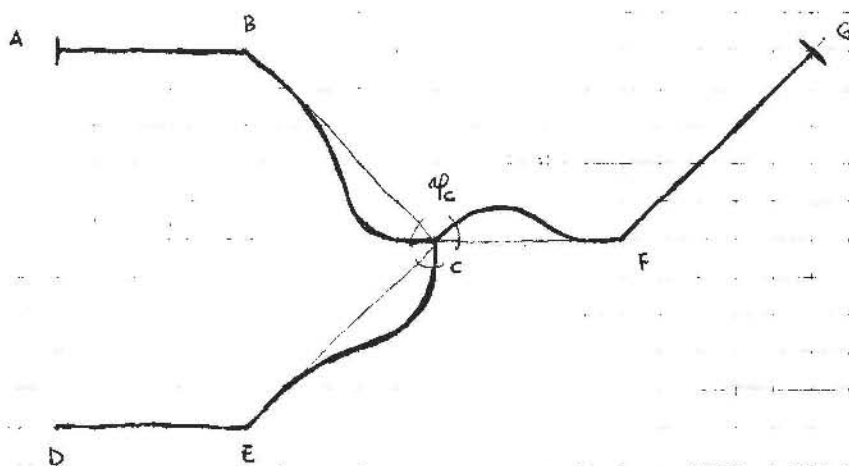
- $[\delta_2 \neq 0, \delta_1 = \varphi_B = \varphi_C = \varphi_E = \varphi_F = 0]$



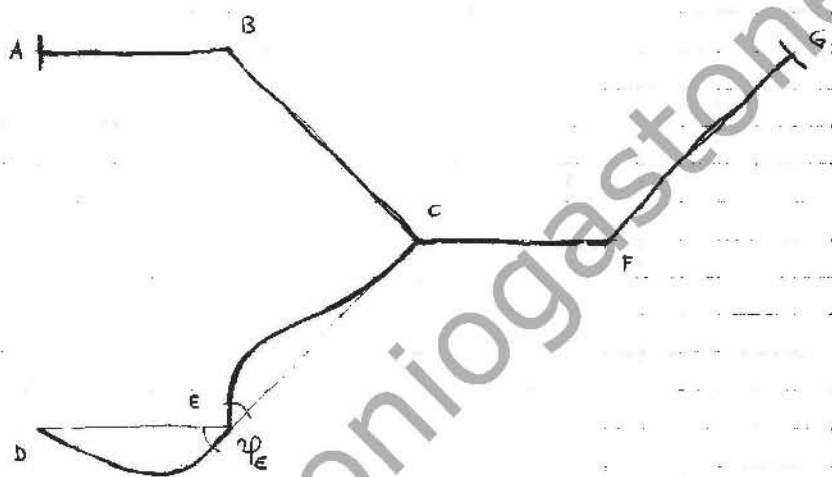
- $[\varphi_B \neq 0, \delta_1 = \delta_2 = \varphi_C = \varphi_E = \varphi_F = 0]$



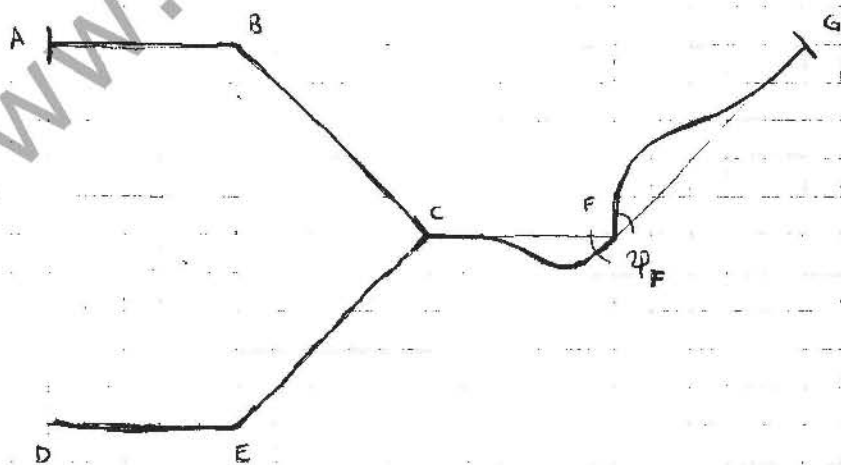
$$\bullet [\varphi_C \neq 0, \delta_1 = \delta_2 = \varphi_B = \varphi_E = \varphi_F = 0]$$



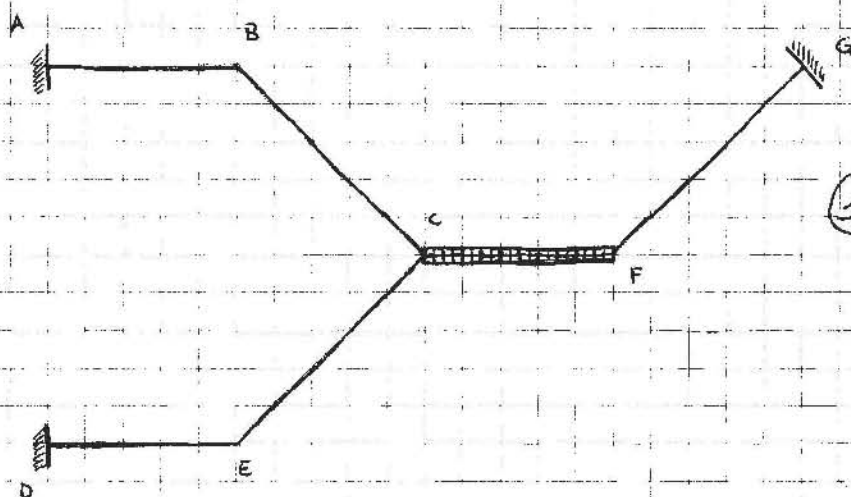
$$\bullet [\varphi_E \neq 0, \delta_1 = \delta_2 = \varphi_B = \varphi_C = \varphi_F = 0]$$



$$\bullet [\varphi_F \neq 0, \delta_1 = \delta_2 = \varphi_B = \varphi_C = \varphi_E = 0]$$



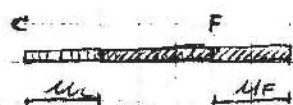
2



12 Parametri Iniziali

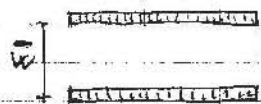
8 Spost.
4 Rotaz.

- Tetto CF Indeformabile [Riduce i parametri indipendenti a 9]



$$u_C = u_F = \bar{u}$$

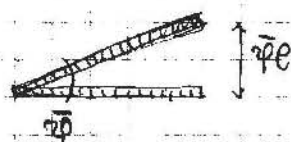
6 Spost.
3 Rot.



$$w_C = \bar{w}$$

$$w_F = \bar{w} - \bar{\varphi}e$$

$$\varphi_C = \varphi_F = \bar{\varphi}$$



- Indeformabilità Anzile [5 aste deformabili \rightarrow 4 parametri indipend.]

10 Spost.
3 Rot.

$$(\Delta e)_{AB} = 0 \rightarrow u_B = 0$$

$$(\Delta e)_{DE} = 0 \rightarrow u_E = 0$$

$$(\Delta e)_{FG} = 0 \rightarrow -\frac{u_F}{\sqrt{2}} + \frac{w_F}{\sqrt{2}} = 0 \rightarrow u_F = w_F \rightarrow w_F = \bar{u} \rightarrow w_F = \bar{w} - \bar{\varphi}e$$

$$\bar{w} = \bar{u} + \bar{\varphi}e$$

$$(\Delta e)_{BC} = 0 \rightarrow -\frac{w_B}{\sqrt{2}} + \frac{u_C}{\sqrt{2}} + \frac{w_C}{\sqrt{2}} = 0 \rightarrow w_B = u_C + w_C$$

$$(\Delta e)_{EC} = 0 \rightarrow \frac{w_E}{\sqrt{2}} + \frac{u_E}{\sqrt{2}} - \frac{w_C}{\sqrt{2}} = 0 \rightarrow w_E = -u_C + w_C$$

$$\begin{cases} u_B = 0 \\ w_B = 2\bar{u} + \bar{\varphi}e \end{cases}, \begin{cases} u_C = \bar{u} \\ w_C = \bar{u} + \bar{\varphi}e \end{cases}$$

$$\Rightarrow w_C = \bar{u} + \bar{\varphi}e$$

$$\Rightarrow w_B = \bar{u} + \bar{u} + \bar{\varphi}e \Rightarrow w_B = 2\bar{u} + \bar{\varphi}e$$

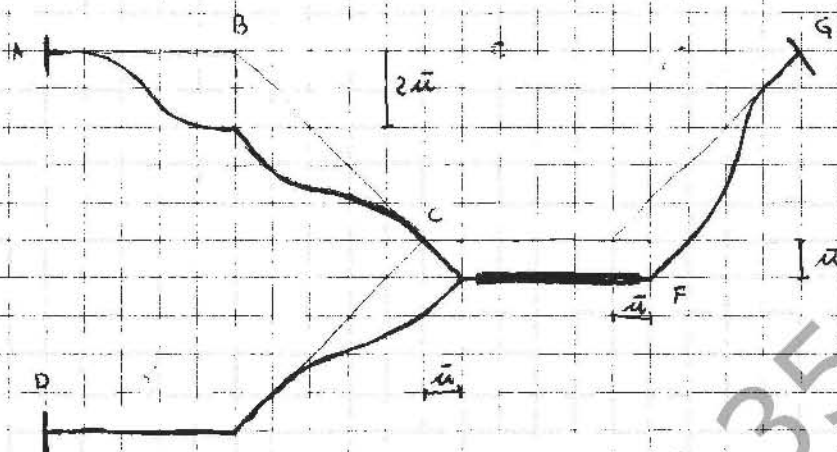
$$\Rightarrow w_E = -\bar{u} + \bar{u} + \bar{\varphi}e \Rightarrow w_E = \bar{\varphi}e$$

$$\begin{cases} u_E = 0 \\ w_E = \bar{\varphi}e \end{cases}, \begin{cases} u_F = \bar{u} \\ w_F = \bar{u} \end{cases}$$

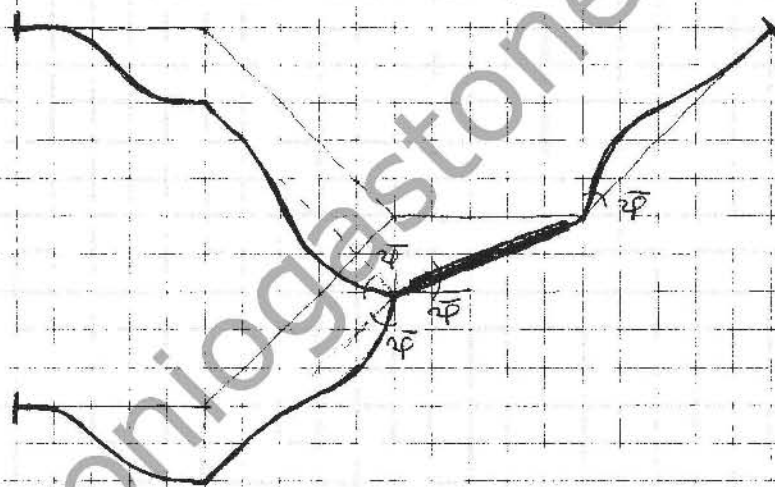
Riepilogo \rightarrow Parametri cinematici indipendenti $\rightarrow \underline{u} = [\bar{u}, \bar{\varphi}, \varphi_B, \varphi_E]$

- Cinematiche Fondamentali

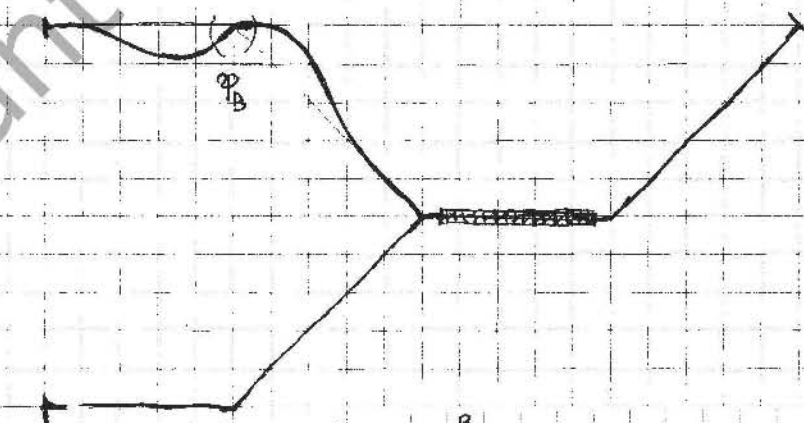
$$\bullet [\bar{\mu} \neq 0, \quad \bar{\Psi} = \Psi_3 = \Psi_E = 0]$$



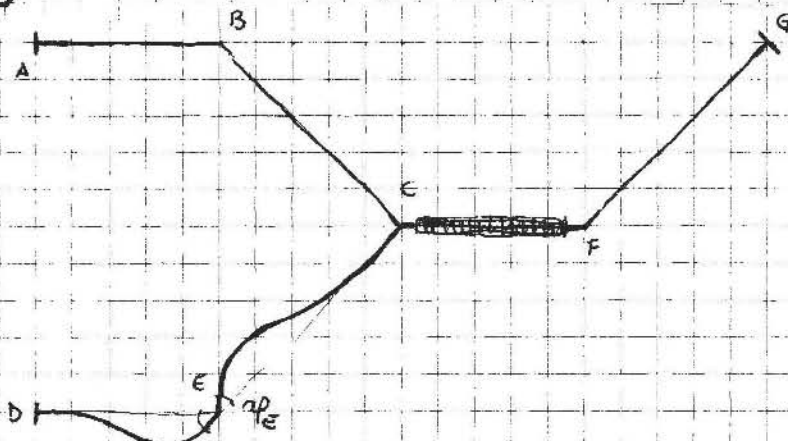
$$\bullet [\bar{\Psi} \neq 0, \bar{u} = \varphi_B = \varphi_E = 0]$$



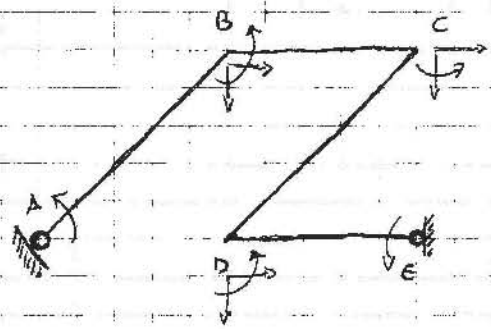
$$\bullet [\varphi_B \neq 0, \bar{\mu} = \bar{\varphi} = \varphi_E = 0]$$



$$\bullet [\varphi_E \neq 0, \bar{\mu} = \bar{\varphi} = \varphi_B = 0]$$



3



11

Parametri cinematici { 6 Spost. 5 Rotaz.

- Indeformabilità Anale [4 vincoli sugli spostamenti] \rightarrow 2 Spost. Indip.

$$(A\epsilon)_{BC} = 0 \rightarrow -u_B + u_C = 0 \rightarrow u_B = u_C \quad \left\{ \begin{array}{l} u_B = \delta_1 \\ u_C = \delta_1 \end{array} \right.$$

$$(A\epsilon)_{AB} = 0 \rightarrow \frac{u_B}{\sqrt{2}} - \frac{w_B}{\sqrt{2}} \rightarrow u_B = w_B \quad \left\{ \begin{array}{l} w_B = \delta_1 \end{array} \right.$$

$$(A\epsilon)_{DE} = 0 \rightarrow u_D = 0$$

$$(A\epsilon)_{CD} = 0 \rightarrow \frac{u_C}{\sqrt{2}} - \frac{w_C}{\sqrt{2}} + \frac{w_D}{\sqrt{2}} = 0 \rightarrow u_C - w_C + w_D = 0, \quad \left\{ \begin{array}{l} w_D = \delta_2 \\ w_C = \delta_1 + \delta_2 \end{array} \right.$$

Riepilogo

$$\left\{ \begin{array}{l} u_B = \delta_1 \\ w_B = \delta_1 \end{array} \right. ; \left\{ \begin{array}{l} u_C = \delta_1 \\ w_C = \delta_1 + \delta_2 \end{array} \right. ; \left\{ \begin{array}{l} u_D = 0 \\ w_D = \delta_2 \end{array} \right.$$

- Parametri Indipendenti Condensazione Rotazioni

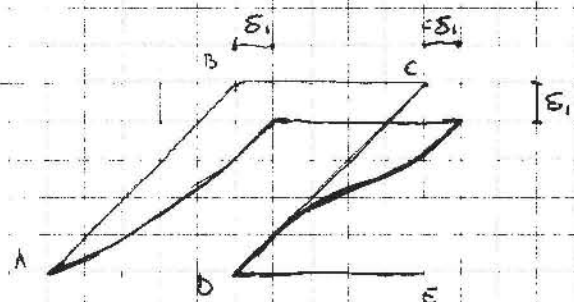
$$\varphi_A = -\frac{\varphi_B}{2}, \quad \varphi_E = -\frac{\varphi_D}{2}$$

- Parametri Cinematici Indipendenti Finali

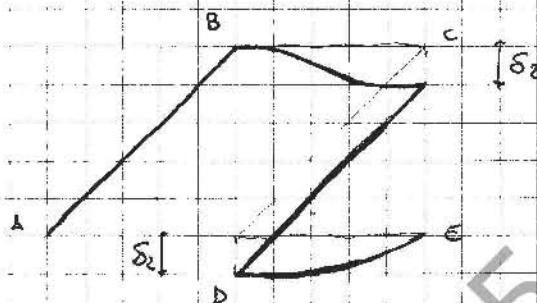
$$u \equiv \{ \delta_1, \delta_2, \varphi_B, \varphi_C, \varphi_D \}$$

- Cinematiche Fondamentali

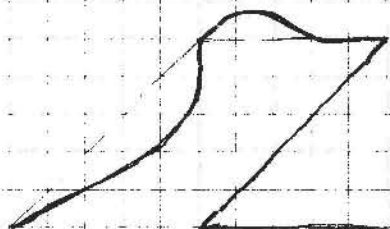
$$\bullet [\delta_1 \neq 0, \delta_2 = \varphi_B = \varphi_C = \varphi_D = 0]$$



$$\bullet [\delta_2 \neq 0, \delta_1 = \varphi_B = \varphi_C = \varphi_D = 0]$$



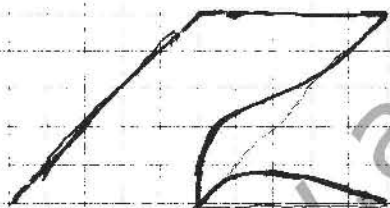
$$\bullet [\varphi_B \neq 0, \delta_1 = \delta_2 = \varphi_C = \varphi_D = 0]$$



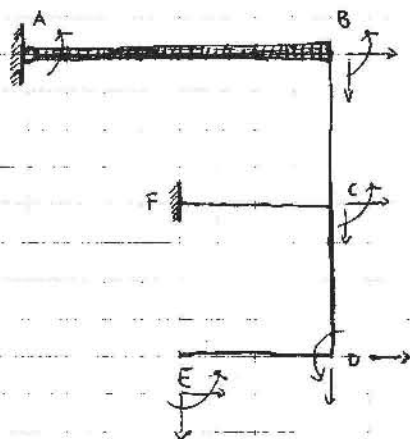
$$\bullet [\varphi_C \neq 0, \delta_1 = \delta_2 = \varphi_B = \varphi_D = 0]$$



$$\bullet [\varphi_D \neq 0, \delta_1 = \delta_2 = \varphi_B = \varphi_C = 0]$$



4



13 Parametri cinematici insidiati

8 spostamenti
5 Rotazioni

- Corpo Rigido → 10 { 6 Spost
4 Rot

- Indef. Anzile → 6 { 2 Spost
4 Rot

- Indeformabilità assiale [4 aste deformabili]

$$(\Delta e)_{AB} = 0 \rightarrow u_B = 0$$

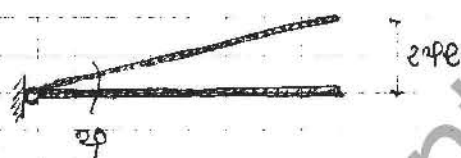
$$(\Delta e)_{FC} = 0 \rightarrow u_C = 0$$

$$(\Delta e)_{BC} = 0 \rightarrow -w_B + w_C = 0 \rightarrow w_B = w_C$$

$$(\Delta e)_{CD} = 0 \rightarrow -w_C + w_D = 0 \rightarrow w_C = w_D$$

$$(\Delta e)_{ED} = 0 \rightarrow -u_E + u_D = 0 \rightarrow u_E = u_D$$

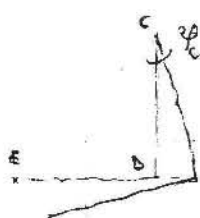
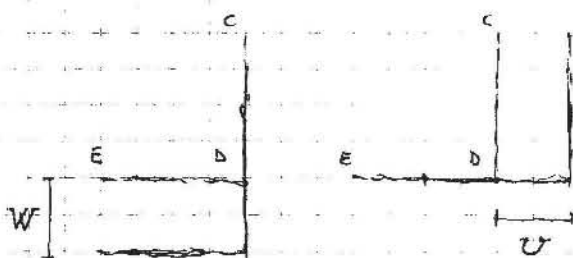
- Corpo Rigido incernierato AB



$$\begin{cases} u_A = u_B = 0 \\ w_A = 0 \\ w_B = -2\bar{\varphi} \\ \varphi_A = \varphi_B = \bar{\varphi} \end{cases}$$

- Considerazioni estremo libero

I punti E e D, non sono vincolati, quindi i loro spostamenti sono legati a quelli del punto C.



$$\begin{cases} u_D = u + \bar{\varphi} w \\ w_D = w \\ u_E = u + \bar{\varphi} w \\ w_E = w + \bar{\varphi} w \end{cases}$$

$$\begin{cases} u_C = u \Rightarrow u = 0 \\ w_C = w \end{cases}$$

$$\begin{cases} M_D = \varphi_c \ell \\ W_D = W = W_c \end{cases} ; \begin{cases} M_E = \varphi_c \ell \\ W_E = W_c + \varphi_c \ell \end{cases}$$

Ricordando le relazioni di corpo rigido si ha:

$$\begin{cases} M_A = 0 \\ W_A = 0 \\ \varphi_A = \bar{\varphi} \end{cases} ; \begin{cases} M_B = 0 \\ W_B = -2\bar{\varphi}\ell \\ \varphi_B = \bar{\varphi} \end{cases} ; \begin{cases} M_C = 0 \\ W_C = -2\bar{\varphi}\ell \\ \varphi_C = \varphi_c \end{cases} ; \begin{cases} M_D = \varphi_c \ell \\ W_D = W_C = -2\bar{\varphi}\ell \\ \varphi_D = \varphi_c \end{cases} ; \begin{cases} M_E = \varphi_c \ell \\ W_E = -2\bar{\varphi}\ell + \varphi_c \ell \\ \varphi_E = \varphi_c \end{cases}$$

$$\underline{M} = [\bar{\varphi}, \varphi_c]$$

Cinematiche Fondamentali

$\bar{\varphi} \neq 0$

